Vacuum Channeling Radiation by Relativistic Electrons in a Transverse Field of a Laser-Based Bessel Beam

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Relativistic electrons counterpropagating through the center of a radially polarized J_1 optical Bessel beam in vacuum will emit radiation in a manner analogous to the channeling radiation that occurs when charged particles traverse through a crystal lattice. However, since this interaction occurs in vacuum, problems with scattering of the electrons by the lattice atoms are eliminated. Contrary to inverse Compton scattering, the emitted frequency is also determined by the amplitude of the laser field, rather than only by its frequency. Adjusting the value of the laser field permits the tuning of the emitted frequency over orders of magnitude, from terahertz to soft X rays. High flux intensities are predicted ($\sim$100 MW/cm^2). Extended interaction lengths are feasible due to the diffraction-free properties of the Bessel beam and its radial field, which confines the electron trajectory within the center of the Bessel beam.

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Channeling radiation was predicted by Kumachov [1] to occur when a beam of relativistic positrons is launched almost parallel to the symmetry planes of a solid-state crystal. Classically, this phenomenon can be understood in terms of the charged particle bouncing back and forth between the atomic planes. This transverse oscillation causes the particle to emit radiation. During the early 1980s, Andersen [2] and Klein [3] investigated this process experimentally. A thorough review of activity during these early years was compiled by Bazylev and Zhevago [4].

Quantum mechanically, in the transverse direction, the particle may be conceived to move in an harmonic oscillator [5]. A positron impinging parallel to the symmetry plane of the crystal populates only the lowest eigenstate and, as such, will not radiate. However, when its trajectory is tilted, it also populates the upper states. Spontaneous radiation is emitted as it drops from an upper to a lower energy state.

Throughout the years it has been suggested to replace the crystalline lattice with a macroscopic transverse static field [6] or a superposition of two intersecting laser beams [7]. In the former case, the coupling of the electrons with the radiation is relatively weak for typically available fields, whereas in the latter case, the interaction length is limited due to the small diameters of the crossing laser beams, the need for tight focusing, and diffraction effects (see [8]).

In this Letter we present a new channeling radiation paradigm in which a counterpropagating radially polarized J_1 Bessel beam (BB), see the top frame in Fig. 1, plays a role analogous to that of the lattice. Contrary to the aforementioned alternative schemes for replacing the crystal lattice with an optical lattice, usage of a Bessel beam permits long interaction lengths, which are not limited by the usual diffraction of focused laser beams, and which do not require high laser intensities. Moreover, as will be shown, the electrons bounce back and forth (see inset in Fig. 1) due to the radial force associated with the BB profile, and emit radiation with a frequency proportional to the amplitude of the BB field. The same mechanism responsible for the generation of radiation also facilitates the confinement of the e-beam over the entire length of the BB. It should be mentioned that *coherent* channeling radiation in vacuum is also possible, but this is beyond the scope of this Letter.

Consider a "hollow" J_1 BB (see the main frame in Fig. 1) where the peak of the first lobe is at radius R_{laser} and the e-beam radius R_b is such that $R_b < R_{\mathrm{laser}}$; this is in order to ensure that the electrons are propagating within the BB where the radial force exerted by the potential is linear in r. This region is between the vertical dashed lines depicted in Fig. 1. The laser field components may be derived from the longitudinal component of the electric field $E_z = E_0\cos[\omega_0(t + z/v_{\mathrm{ph}})]$, where E_0 is the amplitude of the laser field, ω_0 is the laser frequency, and v_{ph} is the phase velocity of the light wave.

The BB propagates in the opposite direction to the electrons moving along $z \sim vt$ and, according to Maxwell's stress tensor, the time-averaged radial force density it exerts is

$$\langle f_r\rangle_T = -\frac{\epsilon_0}{2}\left(\frac{\omega_0}{c}E_0\sin\theta_0\right)^2 r, \tag{1}$$

where ϵ_0 is permittivity and θ_0 is the angle of the light ray relative to the z axis, such that $v_{\mathrm{ph}} = c/\cos\theta_0$. Denoting by n_{el} the electron density that is exposed to this BB, the transverse components of the equation of motion read

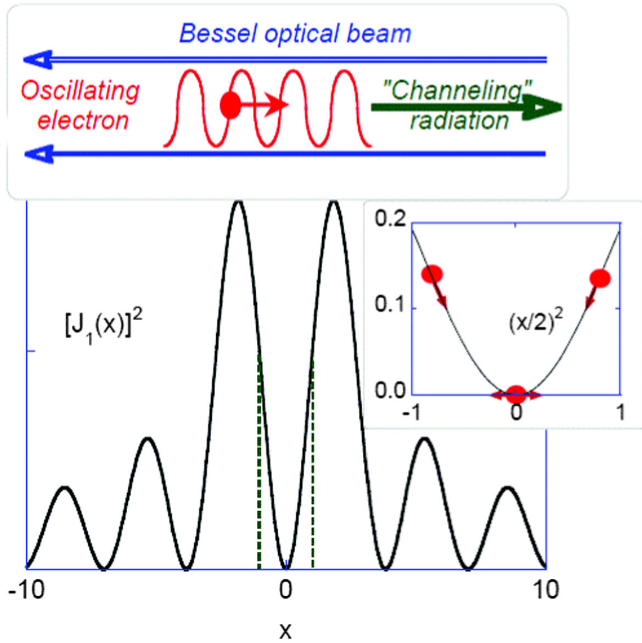

FIG. 1 (color online). A radially polarized $J_1(x)$ Bessel optical beam, the intensity cross section of which is illustrated in the main frame, counterpropagates against a narrow beam of electrons. For stable transverse motion, the diameter of the electron beam is smaller than the distance between the first lobes of the $J_1(x)$ Bessel beam; see vertical dashed (green) lines, $|x| < 1.841$. In this range, the transverse motion of the electrons is determined by the initial conditions and the effective potential, illustrated in the inset at right. As the electrons oscillate transversely back and forth, they generate "channeling" radiation.

$$mn_{el}\frac{d}{dt}\gamma\frac{d}{dt}\begin{pmatrix}x\\y\end{pmatrix} = -\frac{\epsilon_0}{2}\left(\frac{\omega_0}{c}E_0\sin\theta_0\right)^2\begin{pmatrix}x\\y\end{pmatrix}. \quad (2)$$

Ignoring energy exchange, the electrons oscillate in the transverse direction with an angular frequency

$$\Omega^2 \equiv \omega_0^2\frac{(\epsilon_0/2)(E_0\sin\theta_0)^2}{n_{\rm el}\gamma mc^2} \equiv \frac{1}{\gamma}\Omega_0^2, \quad (3)$$

implying that the average contribution of the transverse motion to the kinetic energy is proportional to $\beta_\perp^2 = (\Omega R_b/2c)^2$. To gain an idea of the magnitude of Ω, assume a laser wavelength of $\lambda_0 = 1\ \mu$m, $E_0 = 1$ GV/m, $\gamma = 600$, $\sin\theta_0 = 0.1$, $R_b = 100\ \mu$m, and $n_{\rm el} = 10^9/[\pi R_b^2 \times 1\ {\rm cm}]$; then, $\Omega/2\pi \sim 5$ THz $\rightarrow 60\ \mu$m.

There are two noteworthy observations that distinguish our scheme from other efforts. First, as mentioned, BBs possess the inherent property of maintaining a constant transverse intensity distribution over distances not limited by the Rayleigh length of regular focused laser beams. Second, the "lattice potential" created by the BB radial field exerts a confining force on the electrons, as revealed by Eq. (1), that helps maintain the electron trajectory within the channel formed by the BB.

A first-order estimate of the maximum normalized emittance $\epsilon_N^{\rm max}$, which still results in a stable trajectory, can be provided by noting, as mentioned in the Fig. 1 caption, that the e-beam diameter must fit within the BB potential well and that the β function associated with this beam must, therefore, be c/Ω. Hence, $\epsilon_N \ll \epsilon_N^{\rm max} \equiv R_b^2 c/\Omega$. For the parameters given in the previous paragraph, $\epsilon_N^{\rm max} = 10^3$ mm mrad, which is easily satisfied by typical cathodes.

To calculate the incoherent radiation emitted by the transverse oscillation of the electrons traversing through the hollow laser beam, we designate $\beta_\parallel$ as the ratio of the longitudinal electron velocity to the speed of light and define $\gamma_\parallel^2 \equiv 1/(1-\beta_\parallel^2) \simeq \gamma^2/[\bar{\Omega}_0^2+1]$, where $\bar{\Omega}_0 \equiv \Omega_0 R_b\sqrt{\gamma}/2c$. Since, obviously, $\gamma_\parallel \geq 1$, to ensure *stable* motion whereby the electrons stay confined within the center of the BB, we must have $\gamma - \gamma^{-1} > (\Omega_0 R_b/2c)^2$; this implies that for a given e-beam energy γ there is a limit to the maximum value for Ω_0^2. Explicitly, according to the right-hand side of the last constraint, for a given γ, the ratio of the energy density of the laser beam ($\propto E_0^2$) to the e-beam energy density ($\propto n_{\rm el}$) must be less than $\gamma - 1/\gamma$.

With the oscillation frequency and the preliminary stability condition established, we may proceed to an assessment of the amount of incoherent emission. Defining the interaction length D as the distance the electrons propagate through the BB, the angular and frequency energy spectrum as well as the total energy $w_{\rm tot}$ radiated by $N_{\rm el}$ electrons are

$$\frac{dw}{d\omega d\Omega'} \propto w_{\rm tot}{\rm sinc}^2\left\{\frac{D}{2c}\left[\omega\left(\frac{1}{\beta_\parallel}-\cos\theta\right)-\frac{\Omega}{\beta_\parallel}\right]\right\},$$
$$w_{\rm tot} = N_{\rm el}\frac{e^2}{4\pi\epsilon_0}\left(\frac{\Omega}{c}\gamma_\parallel\right)^4\frac{\beta_\parallel}{2}(\pi R_b^2 D), \quad (4)$$

where $d\Omega' = 2\pi\sin\theta d\theta$. It is tacitly assumed that $\gamma \gg 1$ and that the radiation is emitted in a narrow angle, i.e., $\theta \ll \pi$.

According to Eq. (4), the peak frequency occurs in the forward direction when the argument of the sinc function is zero; thus,

$$\omega_{\theta=0} \simeq \frac{2c}{R_b}\gamma\frac{2\bar{\Omega}_0}{\bar{\Omega}_0^2+1}, \quad (5)$$

and it is important to note that this frequency is determined by the *amplitude* of the BB as revealed in Eq. (3). Moreover, it has a maximum when $\bar{\Omega}_0 = 1$; this, when combined with the condition for stable transverse motion specified above, implies that γ should be greater than $\sqrt{2}$. Equivalently, $(\Omega_0 R_b/2c)^2 = 1/\gamma < 1/\sqrt{2}$, or, explicitly, for a given electron density, the injected laser energy density is limited to $W_B < W_B^{(\rm max)} \equiv (1/\sqrt{2})n_{\rm el}mc^2(\omega_0 R_b/2c)^{-2}$. Vice versa, at a given BB energy density, for stable transverse motion, the electron density must exceed a minimum value,

$$n_{\mathrm{el}} > n_{\mathrm{el}}^{(\mathrm{min})} \equiv \sqrt{2}\frac{W_B}{mc^2}\left(\frac{\omega_0}{c}\frac{R_b}{2}\right)^2, \quad (6)$$

wherein $W_B \equiv \epsilon_0(E_0 \sin\theta_0)^2/2$ is proportional to the energy density associated with the BB in the e-beam region.

One should not be misled by the fact that, according to Eq. (6), the electron density has only a lower limit. To establish the upper limit, we need to keep in mind that if the electron density is too high, the transverse emittance will become space-charge dominated. In order to avoid this situation, we require that the expelling space-charge force density that acts near the e-beam envelope $e^2 n_{\mathrm{el}}^2 R_b/\epsilon_0\gamma_\parallel^2$ be smaller than the radial force density exerted by the BB [Eq. (1)]; consequently,

$$n_{\mathrm{el}} < n_{\mathrm{el}}^{(\mathrm{max})} \equiv \frac{1}{2}\left[\sqrt{n_1^2 + 4n_2^2} - n_1\right] \simeq n_2, \quad (7)$$

wherein $n_1 \equiv \gamma W_B(\omega_0 R_b/2c)^2/mc^2$ and $n_2^2 \equiv W_B\gamma^2\epsilon_0(ec/\omega_0)^{-2}$. For most cases of interest, $n_2 \gg n_1$; therefore, $n_{\mathrm{el}}^{(\mathrm{max})} \simeq n_2$.

Now we return to the radiation characteristics. The bandwidth $\Delta\omega$ of the emitted incoherent radiation has several contributions: (i) the finite interaction length D, (ii) the angular spread $\Delta\theta$, and (iii) the energy spread $\Delta\gamma$. For the resonance occurring at the peak of the sinc function, the full width at half maximum occurs when the argument of sinc is $\simeq\pi/2$. Hence, assuming zero angular and energy spread, the relative bandwidth due to finite interaction length is $(\Delta\omega/\omega)_{\theta,\Delta\theta,\Delta\gamma=0} \simeq 2\pi\sqrt{\gamma_0}/(\Omega_0 D/c)$. For comparison, in a free-electron laser (FEL) [9], the relative bandwidth is $(\Delta\omega/\omega)^{\mathrm{FEL}}_{\theta,\Delta\theta,\Delta\gamma=0} \simeq \lambda_w/D$, which is virtually identical to our case; λ_w is the wiggler's period and D is the wiggler's length.

In the same way, assuming zero energy spread and "infinite" interaction length, the relative bandwidth due to the angular spread of the electrons is $(\Delta\omega/\omega)_{\theta,\Delta\gamma=0,D\to\infty} \simeq 2\gamma_\parallel^2\Delta\theta^2$, a result which is identical to a FEL. Finally, for the energy spread effect we find $(\Delta\omega/\omega)_{\theta,\Delta\theta=0,D\to\infty} \simeq (3/2)\Delta\gamma/\gamma_0$; the corresponding value for a FEL is $4\Delta\gamma/\gamma_\parallel$. Hence, the overall relative bandwidth is the quadrature sum

$$\frac{\Delta\omega}{\omega} \simeq \sqrt{\left(\frac{2\pi c\sqrt{\gamma_0}}{\Omega_0 D}\right)^2 + (2\gamma_\parallel^2\Delta\theta^2)^2 + \left(\frac{3\Delta\gamma}{2\gamma_0}\right)^2}. \quad (8)$$

Equation (8) indicates that achieving narrow band emission favors a long interaction length D and relatively low e-beam energy. For example, assuming $D = 10$ cm, $\lambda_{\mathrm{laser}} = 532$ nm, and a 76-MeV e-beam, such as the one available at the Brookhaven National Laboratory Accelerator Test Facility (ATF) [10], $(\Delta\omega/\omega)_{\theta,\Delta\theta,\Delta\gamma=0} \simeq 2.3\times10^{-4}$. For $\Delta\theta = 5$ mrad, $(\Delta\omega/\omega)_{\theta,\Delta\gamma=0,D\to\infty} \simeq 5.7\times10^{-4}$. The ATF e-beam has a typical energy spread of 0.03% [11]; thus, $(\Delta\omega/\omega)_{\theta,\Delta\theta=0,D\to\infty} \simeq 3.0\times10^{-6}$. Therefore, substituting these values into Eq. (8) gives a relative bandwidth of 6.2×10^{-4}. The fact that this narrow bandwidth was achieved with an interaction length of only 10 cm attests to the potential compactness of our scheme. Note this bandwidth is over 2 orders of magnitude smaller than the spontaneous emission from an undulator, and is comparable to a FEL. It is also comparable to the bandwidth of inverse Compton scattering (ICS) [12].

A similar approach may be employed to determine the impact of the various quantities that determine the effective wiggler ($\bar{\Omega}_0$) impact on the relative bandwidth

$$\frac{\Delta\omega}{\omega} \simeq \frac{|1-\bar{\Omega}_0^2|}{1+\bar{\Omega}_0^2}\sqrt{\left(\frac{\Delta E}{E}\right)^2 + \left(\frac{\Delta\omega_0}{\omega_0}\right)^2 + \left(\frac{\Delta n_{\mathrm{el}}}{n_{\mathrm{el}}}\right)^2}. \quad (9)$$

Note that the impact of the channel on the bandwidth is small if the condition for maximum frequency [in Eq. (5)] is satisfied ($\bar{\Omega}_0 \simeq 1$). Commercial diode-pumped, mode-locked Nd:YAG lasers are available with pulse-to-pulse output power stabilities of $\leq 0.2\%$, implying $\Delta E/E \leq 0.001$, and they have a typical frequency linewidth of order 1 cm^{-1}, corresponding to $\Delta\omega_0/\omega_0 \sim 10^{-4}$. The microbunch charge stability from photocathode-driven linacs is ~0.4% [13]; hence, $\Delta n_{\mathrm{el}}/n_{\mathrm{el}}$ is 4×10^{-3}. Thus, a typical value for the radical in Eq. (9) is $\sim 4\times10^{-3}$.

The wavelength λ_{rad} [based on Eq. (5)] and flux S of the emitted channeling radiation in the forward direction ($\theta = 0$) are plotted in Figs. 2(a) and 2(b), respectively, as a function of the laser field for three different e-beam energies. Flux S is calculated from Eq. (4) as the energy w_{tot} emitted from the e-beam over a cross section πR_b^2 and length Δ_z, i.e., $S = w_{\mathrm{tot}}c/\pi R_b^2\Delta_z$. The other parameter values for the plots shown in Fig. 2 are $R_b = 20$ μm, $D = 0.10$ m, $N_{\mathrm{el}} = 1.87\times10^{10}$ (3 nC), $\Delta_z = 3$ mm (electron bunch duration = 10 ps), $\sin\theta_0 = 0.10$, and $\lambda_{\mathrm{laser}} = 0.532$ μm. For an indication of the required laser peak power, consider a Gaussian laser beam in vacuum, $E = E_0\exp[-(r/w_0)^2]$, with waist $w_0 \sim 100$ μm, which is larger than the e-beam radius of 20 μm; then, $E_0 = \sqrt{2P_{\mathrm{laser}}/\pi\epsilon_0 c w_0^2}$, or, explicitly, if $E_0 = 10^9$ V/m, then the laser peak power is of the order $P_{\mathrm{laser}} \sim 40$ MW. This peak power is readily available from solid-state lasers.

As seen in Fig. 2(a), a wavelength-tuning range of many orders of magnitude is possible by simply varying the laser intensity. Depending on the e-beam energy, this range can be from the terahertz (THz) regime for low-energy e-beams down to soft X rays for high-energy e-beams (e.g., several GeV). Each curve displays a

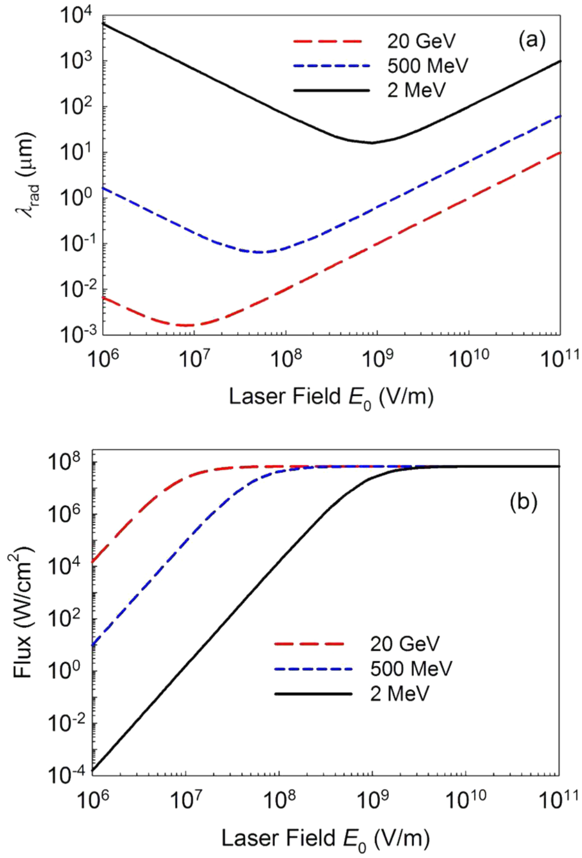

FIG. 2 (color online). Predicted emission characteristics in the forward direction for channeling radiation in vacuum as a function of the Bessel beam field (see text for values of other parameters). (a) Emission wavelength. (b) Emitted flux.

minimum wavelength that can be achieved for a given e-beam energy. This minimum can be readily understood in terms of the maximum that $\omega_{\theta=0}$ reaches [see Eq. (5)] if $\bar{\Omega}_0 = \sqrt{\gamma}\Omega_0 R_b/2c = 1$, in which case $\omega_{\theta=0}^{\max} = 2c\gamma/R_b$. For a 500-MeV e-beam and $R_b = 20\ \mu$m, this implies $\lambda_{\theta=0}^{\min} = 64$ nm, which agrees with the minimum wavelength indicated in Fig. 2(a) for the 500-MeV e-beam.

In Eq. (4), the energy spectral density was determined in terms of the total energy emitted, w_{tot}. We have calculated the total number of photons emitted by a single electron during the process. When operating at $\omega_{\theta=0}^{\max}$, the number of photons emitted per electron is $(N_{\mathrm{ph}}/N_{\mathrm{el}}) \simeq 2\pi\alpha D/\gamma_0 R_b$; $\alpha \equiv e^2/4\pi\epsilon_0\hbar c$ is the fine-structure constant. As in the case of the relative bandwidth, it is interesting to contrast this with the corresponding ratio for FELs. For example, a 2-MeV beam would produce 57 photons per electron for $D = 0.1$ m. However, for a FEL of similar length, only ~0.05 photons per electron are produced, assuming $B_w = 1$ T and $\lambda_w = 1$ cm. This implies that the channeling radiation in vacuum scheme can be more efficient at generating photons for a given e-beam current.

Figure 2(b) demonstrates that high flux is also obtainable. For the parameter values chosen, the flux levels off at 69 MW/cm^2 regardless of the e-beam energy. This leveling off of the flux is expected, since w_{tot} is a function of $\Omega/\gamma_{\|}$ and $\gamma_{\|}$ depends on Ω. Recall that Ω depends also on the laser field E_0. Hence, at sufficiently high values of E_0 relative to the e-beam energy, the flux becomes independent of the laser field. This also implies that high fluxes are possible even at low-energy e-beams, subject to the condition in Eq. (6).

Although the BB counterpropagates with respect to the e-beam as in ICS, our process is fundamentally different from ICS in several ways. First, in our scheme, the on-axis laser field is zero, whereas it is maximized for ICS. In that regard, we are relying on the fact the interaction via ICS is negligible. Second, the electron oscillation is in the transverse direction in a potential well (harmonic oscillator) whose eigenfrequency is proportional to the energy density of the laser. In other words, the emitted photon is determined not just by the frequency of the laser, as it is for ICS, but also by the intensity of the BB. This is a noteworthy characteristic, since it means that one can control the frequency spectrum of the emitted radiation with the laser intensity. Finally, the emitted frequency also varies in a manner inversely proportional to the square root of the density of the electrons, which is unlike ICS.

In conclusion, we have analyzed a novel radiation source that resembles channeling radiation, but which occurs in vacuum as a result of oscillation of electrons in the field of a counterpropagating Bessel beam. Consequently, extended interaction regions, which require only moderate laser peak power, are feasible. Contrary to inverse Compton scattering, the tunability of the emitted radiation is controlled primarily by the amplitude of the Bessel beam.

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