# **Gas-Filled Capillary Model**

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Abstract. We have developed a 1-D, quasi-steady-state numerical model for a gas-filled capillary discharge that is designed to aid in selecting the optimum capillary radius in order to guide a laser beam with the required intensity through the capillary. The model also includes the option for an external solenoid *B*-field around the capillary, which increases the depth of the parabolic density channel in the capillary, thereby allowing for propagation of smaller laser beam waists. The model has been used to select the parameters for gas-filled capillaries to be utilized during the Staged Electron Laser Acceleration – Laser Wakefield (STELLA-LW) experiment.

**Keywords:** Capillary discharge plasma, numerical model, laser wakefield acceleration **PACS:** 41.75.Jv, 52.75.-d, 52.38.Hb, 52.40.Fd

## **INTRODUCTION**

Gas-filled capillary discharges are one effective means for producing plasmas for laser wakefield acceleration (LWFA) and plasma wakefield acceleration (PWFA). Hooker and his colleagues pioneered the development of these gas-filled capillaries [1]. The basic configuration for a gas-filled capillary is illustrated in Fig. 1. Hydrogen gas is injected into the waveguide, which might be centimeters long with an inside diameter of  $\sim$ 1 mm or less. A high-voltage pulse is applied across the electrodes on the ends of the waveguide to ignite the plasma discharge. The hydrogen gas continues to flow out the ends of the capillary into the beamline vacuum.

Gas-filled capillaries can have advantages over ablative capillaries [2], including independent control of the plasma density by varying the gas-fill pressure and longer capillary tube lifetime. However, the gas handling system and dealing with the gas exhausted into the beamline vacuum can make its operation more complicated.

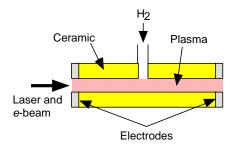


FIGURE 1. Schematic cross-sectional diagram of the basic configuration for a gas-filled capillary.

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In LWFA the capillary serves two important functions. It allows one to adjust the plasma density for optimum performance of the wakefield accelerator. It also guides the laser beam by producing a plasma channel with a parabolic radial density distribution in its center. This channeling is important because it allows the laser intensity inside the plasma to remain constant over lengths much longer than the Rayleigh range.

Usually the value of the on-axis plasma density  $n_e$  is fixed by other constraints, such as the laser beam characteristics. With  $n_e$  chosen, one must decide on the other design parameters for the capillary (e.g., capillary radius) that will result in guiding the laser beam. Guiding occurs when the matched laser beam waist  $W_m$  satisfies the following equation [1],

$$W_m \approx \left(\frac{R_0^2}{\pi r_e \Delta n_e}\right)^{1/4},\tag{1}$$

where  $R_0$  is the channel radius;  $r_e = 2.82 \times 10^{-13}$  cm; and  $\Delta n_e$  is the depth of the density channel (i.e., the difference between the density at r = 0 and  $r = R_0$ ) assuming the density follows a parabolic profile.

The minimum laser beam waist size is often determined by the minimum required laser intensity inside the plasma for a given available laser pulse energy. Thus, Eq. (1) shows that the channel radius is controlled by the depth of the parabolic channel needed to achieve the required value for  $W_{\rm m}$ . A larger depth  $\Delta n_e$  permits operating with a larger capillary radius. This can be important because a larger capillary radius helps reduce the laser field at the capillary wall, thereby minimizing the possibility of damage to the capillary wall. In general, one would like the ratio  $R_0/W_{\rm m}$  to be as large as possible to ensure safe operation with the laser beam.

The depth of the parabolic channel is affected by the plasma formation process, which tends to be complex and influenced by a number of different factors. Therefore, selecting the optimum parameters for the capillary design is nontrivial and requires the help of a model.

A comprehensive, time-dependent, magnetohydrodynamic model for a gas-filled capillary has been developed by Bobrova, *et al.* [3], which agrees well with experimental data. However, for simply designing a capillary for a particular application, a fully time-dependent model is unnecessary. The Bobrova computations indicate a quasi-steady-state is reached fairly quickly (e.g.,  $\approx$ 50-80 ns). This quasi-steady-state lasts as long as the discharge is maintained (e.g.,  $\sim 1 \ \mu$ s). A quasi-static assumption greatly simplifies the model, reducing it to a coupled set of ordinary differential equations solvable by a simple iteration procedure.

This paper describes the results from a quasi-static, 1-D model that was specifically made to aid in the design of a gas-filled capillary discharge. The model has been used to design gas-filled capillaries for the Staged Electron Laser Acceleration – Laser Wakefield (STELLA-LW) experiment [4].

#### **BRIEF DESCRIPTION OF QUASI-STATIC MODEL**

Details of the model are given elsewhere [5]. It is designed to be user-friendly and can be run on a standard desktop PC. Briefly, our model emphasizes simplicity while including a broad range of physics elements. It is based on a 1-D in r quasi-steady-state plasma discharge in hydrogen in which conductive thermal loss balances ohmic heating by the discharge current. A full Braginskii [6] transport model is employed, allowing partial magnetization of both species (ions, electrons) and thermoelectric effects. The model agrees well with the Brobova *et al.* code [3].

The plasma density profile is roughly parabolic over most of the channel radius, but deviates significantly from a parabolic shape near the walls. In fact, the density becomes high near the walls, which helps refract the laser light away from the walls. This can help reduce the probably of laser damage to the walls. Therefore, another feature of our model is it also calculates the self-consistent propagating field structure of the laser beam near the walls. This allows the electric field at the capillary wall to be determined more precisely, enabling a better estimate on the wall laser-damage limit.

Another innovation that we developed was the inclusion of an optional external solenoid B-field surrounding the capillary. The presence of this field permits operation at higher plasma temperatures by reducing the thermal conductivity of the plasma. This in turn helps increase the depth of the parabolic profile. A larger depth allows one to use a larger capillary channel radius, which can have several benefits. These benefits include reducing the laser field on the capillary walls, thereby, decreasing the probably of laser damage to the walls, and easing the fabrication of the capillary.

Besides assuming a quasi-steady-state, the model also assumes: 1) a large aspect ratio between the capillary length and radius, which is usually the case for LWFA and PWFA experiments; 2) upon initiation of the discharge, the plasma adjusts radially, however, once quasi-steady-state is reached, ion motion ceases and the electrons carry all the current; 3) quasi-neutrality, i.e., the ion density equals the electron density, which is a very good assumption except right at the capillary wall; 4) pure hydrogen as the gas; 5) a single plasma temperature, i.e., the ions and electrons have the same temperature; 6) negligible parasitic losses, e.g., radiative power loss is neglected; and 7) classic Braginskii transport is valid [6].

The basic parameters that must be chosen for the capillary design are: 1) radius  $R_0$ , 2) length L, 3) plasma density  $n_e$ , 4) discharge current I, and 5) the matched laser waist  $W_m$ . An additional parameter is the optional external B-field  $B_z$ . The capillary length is typically set by other considerations, such as the dephasing length. It is not considered in the quasi-static model except for the assumption that  $L >> R_0$ . The required plasma density is typically determined by the characteristics of the laser drive pulse, i.e., its pulse duration. Thus, there usually is a target value for  $n_e$  that needs to be obtained. In a gas-filled capillary this can be accomplished by adjusting the amount of gas sent into the capillary. As mentioned, the matched laser waist size is dictated by the amount of laser intensity needed to drive the process. Therefore, the value for  $W_m$  is often preset. This leaves as free parameters for the capillary designer only  $R_0$  and I.

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450	0.89	4	12.854	0.307	1.000	0.300	158.3	1.291	0.943	157.2	2.863	
160	0.11	3.3	18.720	0.306	1.000	0.300	16.6	0.165	0.833	145.1	1.103	
450	0.89	4	12.854	0.307	1.000	0.300	158.3	1.291	0.943	157.2	2.863	

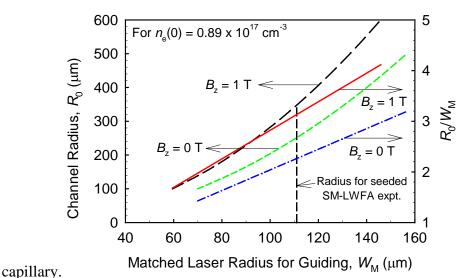
FIGURE 2. Example of input/output spreadsheet for quasi-static model.

Operation of the model typically consists of selecting a value for I and, if available,  $B_z$ , and having the model calculate a value for  $R_0$  for a given value of  $W_m$ . Figure 2 shows an example of the model input/output interface on the computer monitor. The user inputs are in the upper left-hand section. Based on the Brobova model [3], an on-axis electron temperature of order 4-7 eV is a reasonable assumption. The longitudinal electric field of the discharge  $E_z$  affects the value of the discharge current.

With the inputs entered, the "iterate" button is clicked on, which starts the iteration process. At the end of each iteration sequence, the plots as shown in Fig. 2 are updated. These plots include the plasma temperature versus normalized radius, and the pressure and plasma density versus normalized radius. Further clicking on the iterate button normally causes the curves in the plots to converge to a stable solution. Once this state is reached, the simulation is concluded. If the plots begin diverging from a stable solution, then this usually signifies no proper solution exists for the input parameters chosen so the parameters must be modified. The final results for the sheet. At the bottom of the sheet is an example of the results for various model runs.

#### QUASI-STATIC MODELING RESULTS

To illustrate sample results from the quasi-static model, we shall assume the planned parameters for the STELLA-LW experiment. Specifically, for the seeded self-modulated LWFA experiment [7], the LWFA model predicts good performance with  $n_e(r = 0) = 8.9 \times 10^{16}$  cm<sup>-3</sup>. At this density and an electron temperature of 6.17 eV, Fig. 3 shows the required capillary radius  $R_0$  as a function of the matched laser waist  $W_m$  with and without the external solenoidal B-field  $B_z$ . The plasma temperature was chosen based upon the temperatures determined in [3] after reaching quasi-steady-state. Fortunately, the overall results are not sensitive to the plasma temperature. For  $B_z$  we have assumed a value of 1 T, which represents approximately the upper limit for



**FIGURE 3.** Physical channel radius  $R_0$  versus the matched laser radius  $W_M$  for  $n_e(0) = 8.9 \times 10^{16}$  cm<sup>-3</sup> with and without the external magnetic field  $B_z$  for the seeded SM-LWFA experiment.

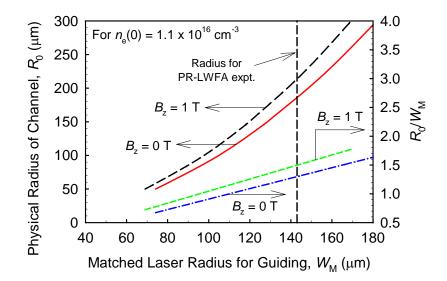
a compact permanent magnet with a small bore that might fit around the capillary.

Plotted on the right side of Fig. 3 is the ratio  $R_0/W_m$ . As explained, one would like this ratio as large as possible to avoid laser damage of the capillary wall. As a rule of thumb, laser damage is generally not an issue if  $R_0/W_m \ge 3$ . However, as mentioned, the model also finds the actual field near the walls. This computation shows the field is smaller at the wall because the density profile increases faster than a parabolic shape near the walls. This implies we may be able to relax the ratio criterion, i.e.,  $R_0/W_m/$ 1.5 may be acceptable.

We see in Fig. 3 a couple of overall trends. A larger matched laser radius allows one to use a larger capillary radius, which increases the ratio  $R_0/W_m$ . For a given  $W_m$ , application of the B-field can significantly increase  $R_0$  and  $R_0/W_m$  over their values without the B-field.

STELLA-LW will be using the TW CO<sub>2</sub> laser located at the Brookhaven National Laboratory Accelerator Test Facility (BNL-ATF). For the laser beam focusing system, the focused beam radius will be approximately 111  $\mu$ m. This beam size corresponds to the matched laser waist as indicated in Fig. 3. Without the B-field, the radius of the channel is 250  $\mu$ m and  $R_0/W_M \sim 2.3$ . With a 1-T B-field, the radius of the channel is 340  $\mu$ m and  $R_0/W_M > 3$ . Although operating with a B-field provides a larger margin of safety, it is likely laser damage will probably not be an issue even without the B-field.

STELLA-LW is also investigating a second LWFA method called pseudo-resonant LWFA (PR-LWFA) [8]. For the conditions at the ATF, this method requires a much lower plasma density of order  $10^{16}$  cm<sup>-3</sup>. At this lower density, the quasi-static model predictions for  $R_0$  versus  $W_M$  are shown in Fig. 4. We see both the capillary radius and the ratio  $R_0/W_M$  are considerably smaller than in Fig. 3. The reason for this is because at the lower density,  $\Delta n_e$  is smaller. Consequently, the parabolic channel is shallower and  $R_0$  becomes comparable in size to  $W_M$  (see Eq. 1).



**FIGURE 4.** Physical channel radius  $R_0$  versus the matched laser radius  $W_M$  for  $n_e(0) = 1.1 \times 10^{16}$  cm<sup>-3</sup> with and without the external magnetic field  $B_z$  for the pseudo-resonant LWFA experiment.

For the PR-LWFA experiment, the matched radius is 143 µm. We see at this radius and without the B-field that  $R_0/W_{\rm M}$  is only 1.3. With the B-field this ratio increases to 1.5. This places it marginally within the range of our  $R_0/W_{\rm m}$  / 1.5 criterion. Nevertheless, the model indicates that laser damage emerges as an important issue at low plasma densities and it may be necessary to utilize a solenoidal B-field.

#### CONCLUSIONS

The quasi-static capillary discharge model is a convenient, easy-to-use tool to assist in the design of a gas-filled capillary. It agrees with a comprehensive, time-dependent model. It includes the option for application of an external B-field to permit using larger diameter capillaries and it provides a more accurate estimation of the laser field magnitude at the capillary wall. Both these features are important to help mitigate possible laser damage at the walls.

The model has been applied towards the design of the capillaries for the STELLA-LW experiment. It indicates that the capillary for the seeded SM-LWFA experiment should not have any serious issues; however, laser damage of the capillary wall for the pseudo-resonant LWFA experiment may be an issue. Utilization of the external Bfield may help alleviate this potential problem.

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